

Order of Magnitude

$f(n)$ is $O(g(n))$ if there exists positive integers a & b such that

$$f(n) \leq a \cdot g(n) \text{ for all } n \geq b.$$

For example:

$$f(n) = n^2 + 100n \text{ \& } g(n) = n^2$$

since $n^2 + 100n \leq 2n^2$ for all $n \geq 100$ ($a=2$ & $b=100$)

but also...

$$f(n) = n^2 + 100n \text{ \& } g(n) = n^3$$

since $n^2 + 100n \leq 2n^3$ for all $n \geq 8$ ($a=2$ & $b=8$)

Both $O(n^2)$ and $O(n^3)$ work. Pick the smallest.

$f(n)$ is asymptotically bounded by $a \cdot g(n)$ at $n \geq b$.

Example: Show $f(n) = cn^k$ is $O(n^k)$

$$cn^k \leq a \cdot n^k \text{ for all } n \geq b$$

If $a=c$, $b=1$ then $cn^k \leq cn^k$ for all $n \geq 1$

For any $f(n) = c_1n^k + c_2n^{k-1} + \dots + c_kn + c_{k+1}$
then $f(n)$ is $O(n^k)$ (Also $f(n)$ is $O(n^{k+j})$ $\forall j \geq 0$).

Seven functional computing times

1. $O(1)$ - constant
2. $O(\log n)$ - logarithmic
3. $O(n)$ - linear
4. $O(n \log n)$ - $n \log n$
5. $O(n^2)$ - quadratic
6. $O(n^3)$ - cubic
7. $O(2^n)$ - exponential

Code

Frequency Count of $x := x + 1$

- | | |
|---|-------|
| a) $x := x + 1$ | 1 |
| b) for $i := 1$ to n do
$x := x + 1$ | n |
| c) for $i := 1$ to n do
for $j := 1$ to n do
$x := x + 1$ | n^2 |

