

# Counting Statements in Loops

```
for (int i = 0; i < n; i++)
  for (int j = 0; j < i; j++)
    x = x + 1;
```

The inner loop statement  $x = x + 1$  is executed once the for the first time of the outer loop, twice for the second time of the outer loop, three times for the third time of the outer loop, and so on, until it is executed  $n$  times for the  $n$ th time of the outer loop. Therefore the total number of times the statement  $x = x + 1$  is executed is:

$$1 + 2 + 3 + \dots + n$$

The sum of this series is computed by lining up the series backwards and adding the two series together.

$$\begin{array}{r} 1 \quad + 2 \quad + 3 \quad + \dots + n \\ + \quad n \quad + n-1 \quad + n-2 \quad + \dots + 1 \\ \hline = \quad (n+1) + (n+1) + (n+1) + \dots + (n+1) \\ = \quad n(n + 1) \quad \text{There are } n \text{ terms, each summing to } (n + 1) \end{array}$$

$$\text{sum} = \frac{n(n+1)}{2}$$